**Supplemental Files**

1. **EngageNY/Eureka Math Lesson**

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Lesson 6: The Order of Operations

Student Outcomes

* Students evaluate numerical expressions. They recognize that in the absence of parentheses, exponents are evaluated first.

Classwork

Opening (5 minutes)

Take a few minutes to review the Problem Set from the previous lesson. Clarify any misconceptions about the use and evaluation of exponents.

Opening Exercise (5 minutes)

Post the following expression on the board, and ask students to evaluate it.

$$3+4×2$$

Ask students to record their answers and report them using personal white boards, cards, or electronic vote devices. Students who arrive at an answer other than $11$ or $14$ should recheck their work.

Discussion (5 minutes)

* How did you evaluate the expression$ 3+4×2$?
	+ *I added* $3+4$ *first for a sum of* $7$*; then, I multiplied* $7×2$ *for a product of* $14$*.*
	+ *I multiplied* $4×2$ *first for a product of* $8$*; then, I added* $ 8+3$ *for a sum of* $8$*.*
* Only one of these answers can be correct. When we evaluate expressions, we must agree to use one set of rules so that everyone arrives at the same correct answer.
* During the last lesson, we said that addition was a shortcut to *counting on*. How could you think about subtraction?
	+ *Subtraction is a shortcut to “counting back.”*
* These were the first operations that you learned because they are the least complicated. Next, you learned about multiplication and division.
* Multiplication can be thought of as repeated addition. Thinking back on Lesson 4, how could you think about division?
	+ *Division is repeated subtraction.*
* Multiplication and division are more powerful than addition and subtraction, which led mathematicians to develop the order of operations in this way. When we evaluate expressions that have any of these four operations, we always calculate multiplication and division before doing any addition or subtraction. Since multiplication and division are equally powerful, we simply evaluate these two operations as they are written in the expression, from left to right.
* Addition and subtraction are at the same level in the order of operations and are evaluated from left to right in an expression. Now that these rules of order of operations are clear, can you go back and evaluate the expression $3+4×2$ as $11$?



* The diagram correctly models the expression$ 3+4×2$.
* With addition, we are finding the sum of two addends. In this example, the first addend is the number $3$. The second addend happens to be the number that is the value of the expression $4×2$; so, before we can add, we must determine the value of the second addend.

Example 1 (5 minutes): Expressions with Only Addition, Subtraction, Multiplication, and Division

Example 1: Expressions with Only Addition, Subtraction, Multiplication, and Division

What operations are evaluated first?

Multiplication and division are evaluated first, from left to right.

What operations are always evaluated last?

Addition and subtraction are always evaluated last, from left to right.

Ask students to evaluate the expressions.

Exercises 1–3

1. $4+2×7$

$$4+14$$

$18$

1. $36÷3×4$

$12×4$

$$48$$

1. $20-5×2$

$$20-10$$

$$10$$

* In the last lesson, you learned about exponents, which are a way of writing repeated multiplication. So, exponents are more powerful than multiplication or division. If exponents are present in an expression, they are evaluated before any multiplication or division.
* We now know that when we evaluate expressions, we must agree to use one set of rules so that everyone arrives at the same correct answer. These rules are based on doing the most powerful operations first (exponents), then the less powerful ones (multiplication and division, going from left to right), and finally, the least powerful ones last (addition and subtraction, going from left to right).
* Evaluate the expression$ 4+6×6÷8$.
	+ $4+(6×6)÷8$

$$4+(36÷8)$$

$$4+4.5$$

$$8.5$$

* Now, evaluate the expression $4+6^{2}÷8$.
	+ $4+(6^{2})÷8$

$$4+(36÷8)$$

$$4+4.5$$

$$8.5$$

* Why was your first step to find the value of$ 6^{2}$?
	+ *Because exponents are evaluated first.*

Example 2 (5 minutes): Expressions with Four Operations and Exponents

Display the following expression.

*Scaffolding:*

Some students may benefit from rewriting the expression on successive lines, evaluating only one or two operations on each line.

Example 2: Expressions with Four Operations and Exponents

$$4+9^{2}÷3×2-2$$

What operation is evaluated first?

Exponents ($9^{2}=9×9=81$)

What operations are evaluated next?

Multiplication and division, from left to right ($81÷3=27;27×2=54$)

What operations are always evaluated last?

Addition and subtraction, from left to right ($4+54=58;58-2=56$)

What is the final answer?

$$56$$

* Evaluate the next two exercises.

While the answers are provided, it is extremely important to circulate to ensure that students are using the correct order of operations to achieve the answer. For example, in Exercise 5, they should show $4^{3}$ first, followed by $2×8$.

Exercises 4–5

1. $90-5^{2}×3$

$$90-25×3$$

$$90-75$$

$$15$$

1. $4^{3}+2×8$

$$64+2×8$$

$$64+16$$

$$80$$

Example 3 (5 minutes): Expressions with Parentheses

* The last important rule in the order of operations involves grouping symbols, usually parentheses. These tell us that in certain circumstances or scenarios, we need to do things out of the usual order. Operations inside grouping symbols are always evaluated first, before exponents and any operations.

Example 3: Expressions with Parentheses

Consider a family of $4$ that goes to a soccer game. Tickets are $\$5.00$ each. The mom also buys a soft drink for$ \$2.00$. How would you write this expression?

$$4×5+2$$

How much will this outing cost?

$$\$22$$

* Here is a model of the scenario:

Consider a different scenario: The same family goes to the game as before, but each of the family members wants a drink. How would you write this expression?

$$4×\left(5+2\right)$$

Why would you add the $5$ and $2$ first?

We need to determine how much each person spends. Each person spends $\$7$; then, we multiply by $4$ people to figure out the total cost.

How much will this outing cost?

$$\$28$$

* Here is a model of the second scenario:

How many groups are there?

$$4$$

What does each group comprise?

$\$5+\$2$, or $\$7$

* The last complication that can arise is if two or more sets of parentheses are ever needed; evaluate the innermost parentheses first, and then work outward.
* Try Exercises 6 and 7.

Exercises 6–7

1. $2+\left(9^{2}-4\right)$

$$2+(81-4)$$

$$2+77$$

$$79$$

1. $2·\left(13+5-14÷\left(3+4\right)\right)$

$$2∙(13+5-14÷7)$$

$$2∙(13+5-2)$$

$$2∙16$$

$$32$$

If students are confused trying to divide $14$ by $3$, reiterate the rule about nested parentheses.

Example 4 (5 minutes): Expressions with Parentheses and Exponents

* Let’s take a look at how parentheses and exponents work together. Sometimes a problem will have parentheses, and the values inside the parentheses have an exponent. Let’s evaluate the following expression.

Place the expression on the board.

* We will evaluate the parentheses first.

Example 4: Expressions with Parentheses and Exponents

$$2×\left(3+4^{2}\right)$$

Which value will we evaluate first within the parentheses? Evaluate.

First, evaluate $4^{2}$, which is $16$; then, add $3$. The value of the parentheses is $19$.

$$2×(3+4^{2})$$

$$2×\left(3+16\right)$$

$$2×19$$

Evaluate the rest of the expression.

$$2×19=38$$

Place the expression on the board:

What do you think will happen when the exponent in this expression is outside of the parentheses?

$$2×(3+4)^{2}$$

Will the answer be the same?

Answers will vary.

Which should we evaluate first? Evaluate.

Parentheses

$$2×(3+4)^{2}$$

$$2×(7)^{2}$$

What happened differently here than in our last example?

The $4$ was not raised to the second power because it did not have an exponent. We simply added the values inside the parentheses.

What should our next step be?

We need to evaluate the exponent next.

$$7^{2}=7×7=49$$

Evaluate to find the final answer.

$$2×49$$

$$98$$

What do you notice about the two answers?

The final answers were not the same.

What was different between the two expressions?

Answers may vary. In the first problem, a value inside the parentheses had an exponent, and that value was evaluated first because it was inside of the parentheses. In the second problem, the exponent was outside of the parentheses, which made us evaluate what was in the parentheses first; then, we raised that value to the power of the exponent.

What conclusions can you draw about evaluating expressions with parentheses and exponents?

Answers may vary. Regardless of the location of the exponent in the expression, evaluate the parentheses first. Sometimes there will be values with exponents inside the parentheses. If the exponent is outside the parentheses, evaluate the parentheses first, and then evaluate to the power of the exponent.

* Try Exercises 8 and 9.

Exercises 8–9

1. $7+\left(12-3^{2}\right)$

$$7+(12-9)$$

$$7+3$$

$$10$$

1. $7+(12-3)^{2}$

$$7+9^{2}$$

$$7+81$$

$$88$$

Closing (5 minutes)

* When we evaluate expressions, we use one set of rules so that everyone arrives at the same correct answer. Grouping symbols, like parentheses, tell us to evaluate whatever is inside them before moving on. These rules are based on doing the most powerful operations first (exponents), then the less powerful ones (multiplication and division, going from left to right), and finally, the least powerful ones last (addition and subtraction, going from left to right).

Lesson Summary

Numerical expression: A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

Statements like “$3+$” or “$3÷0$” are not numerical expressions because neither represents a point on the number line. Note: Raising numbers to whole number powers are considered numerical expressions as well since the operation is just an abbreviated form of multiplication, e.g., $2^{3}=2∙2∙2$.

Value of a numerical expression: The *value of a numerical expression* is the number found by evaluating the expression.

For example: $\frac{1}{3}∙\left(2+4\right)+7$ is a numerical expression, and its value is $9$.

Note: Please do not stress words over meaning here. It is okay to talk about the *number computed*, *computation*, *calculation*, and so on to refer to the value as well.

Exit Ticket (5 minutes)

Name Date

Lesson 6: The Order of Operations

Exit Ticket

1. Evaluate this expression: $39÷\left(2+1\right)-2×\left(4+1\right)$.
2. Evaluate this expression: $12×\left(3+2^{2}\right)÷2-10$.
3. Evaluate this expression: $12×(3+2)^{2}÷2-10$.

Exit Ticket Sample Solutions

1. Evaluate this expression: $39÷\left(2+1\right)-2×\left(4+1\right)$.

$$39÷3-2×5$$

$$13-10$$

$$3$$

1. Evaluate this expression: $12×\left(3+2^{2}\right)÷2-10$.

$$12×\left(3+4\right)÷2-10$$

$$12×7÷2-10$$

$$84÷2-10$$

$$42-10$$

$$32$$

1. Evaluate this expression: $12×(3+2)^{2}÷2-10$.

$$12×5^{2}÷2-10$$

$$12×25÷2-10$$

$$300÷2-10$$

$$150-10$$

$$140$$

Problem Set Sample Solutions

Evaluate each expression.

1. $3×5+2×8+2$

$$15+16+2$$

$$33$$

1. $\left(\$1.75+2×\$0.25+5×\$0.05\right)×24$

$$(\$1.75+\$0.50+\$0.25)×24$$

$$\$2.50×24$$

$\$60.00$

1. $\left(2×6\right)+\left(8×4\right)+1$

$$12+32+1$$

$45 $

1. $\left(\left(8×1.95\right)+\left(3×2.95\right)+10.95\right)×1.06$

$$\left(15.6+8.85+10.95\right)×1.06$$

$$35.4×1.06$$

$$37.524$$

1. $\left(\left(12÷3\right)^{2}-\left(18÷3^{2}\right)\right)×\left(4÷2\right)$

$$(4^{2}-\left(18÷9\right))×(4÷2)$$

$$(16-2)×2$$

$$14×2$$

$$28$$

1. **Master Evaluation Checklist for Archer & Hughes Lesson**

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| **Mathematics Lesson Plan Evaluation Checklist** |
| **Lesson:** | Order of Operations- Archer and Hughes |
| **Grade Level:** | 8th Graders in a beginning algebra class |
| **Standard(s):** |  |
| **Objective(s):** | Skill: Using the first step in the order of operations |
| **Does the lesson include…** | **Rating 1 2 3**1 = missing2 = insufficient3 = sufficientN/A | **Notes and Suggestions for Revision/Adaptation** |
| **1. A clear teacher statement about lesson objectives?** | 3 | The teacher says, *“Today we are going to continue our work with variables and expressions. You are going to learn how parentheses are used in expressions. This knowledge is critical in solving algebraic expressions.”* This statement is made at the beginning of the lesson opening to let students know exactly what they are going to do/learn. |
| **2. Warm-up/review activities that assess student knowledge of critical pre-skills and activate relevant background knowledge?** | 3 | The review covers the definition of ‘variable’ and identifying variables in expressions; as well as the definition of ‘expression’ and identifying expressions. These are critical pre-skills for success with the current lesson. |
| **3. Modeling w/think aloud?**The lesson provides specific and sufficient guidelines for teacher modeling.\*The teacher shows and tells students how to meet a learning objective (i.e., provides clear, step-by-step demonstrations) using clear, concise, mathematically accurate language. | 3 | The teacher shows and tells students exactly how to perform the skill. Example problems include: 5 x (6 + 3) and (5 x 6) + 3. The teacher models how to apply the skill to both problems to highlight the importance of following the order of operations (i.e., if you don’t follow the OoO, you get a different solution). |
| **4. Segmentation of complex skills?**The lesson breaks a complex skill or strategy into smaller instructional units. | 3 | The teacher presents only the first step of the order of operations and uses a range of examples. Presenting all steps at one time may overwhelm students. |
| **5. Sufficient and appropriate instructional examples that are appropriately sequenced and scaffolded?** | 3 | The selection and sequence of instructional examples is intentional and well-planned. For example, the initial model uses the problem: 5 x (6 +3) and the next examples uses the problem (5 x 6) + 3. These are intentionally selected to highlight the importance of using the order of operations correctly. Also note that the ‘math’ involved in evaluating the selected expressions is not overly complex/difficult. Students can use their attention/energy to focus on the process rather than have their WM overloaded by complex computation. |
| **6. Explicit instruction or review on key mathematics vocabulary terms using precise, student-friendly definitions?** | 3 | The opening exercise reviews key vocabulary. |
| **7. Systematically faded supports?**The lesson promotes successful engagement by providing systematically scaffolded practice opportunities. Initial practice opportunities are heavily supported; then based on students’ response, supports are gradually faded to release responsibility and increase student independence. | 3 | The teacher provides 5 guided practice opportunities. For the first practice opportunity, he tells students what to do, step-by-step, as they follow along. For the next two practice opportunities, the teacher asks questions to prompt students to complete each step and solve. For the fourth practice opportunity, the teacher asks some questions to prime the students/get them ready to solve; and for the fifth/final practice opportunity, the teacher reminds students of the strategy and asks them to solve nearly independently. The practice opportunities start at a very highly-guided/highly-supported level and the teacher fades support gradually until the students are solving on their own. |
| **8. Many opportunities for students to respond with feedback AND opportunities for student verbalizations?** | 2 | The teacher asks many questions throughout the lesson opening, model, and guided practice. The questions are effectively planned to keep students engaged and probe their understanding. The teacher uses unison responding to elicit answers/responses from all students.Students are not asked to verbalize their reasoning or to explain their answers to problems. This part of the lesson could be improved with questioning designed to probe students’ thinking. For example, the teacher asks, “*Do we do the operations inside or outside the parentheses first?*” many times. The teacher could add the follow up questions, “*Why?*” and “*What would happen if we did the operation outside of the parentheses first?”* |
| **9. Purposeful practice?**The lesson provides practice opportunities that align with learning objectives. Practice is distributed and cumulative. Practice may provide opportunities to use concrete manipulatives and/or visual representations to model math skills/concepts. | 3 | The practice problems presented during guided practice and independent practice all provide students with an opportunity to practice the exact skill the teacher taught.The independent work may provide distributed practice.Practice does not include visual representations or concrete manipulatives. These could be included. |
| **10. Correction procedures or guidelines to address student misconceptions?** | 2 | The lesson says: *the teacher monitors and provides feedback* many times throughout, but the lesson does not provide specific guidance on how to correct errors or what student misconceptions to look out for. |
| **Final Evaluation:**Rating total (30 = highest possible score) | 28 | **Final Notes/Comments:**This is a great explicit lesson with clear, direct models and purposeful, scaffolded practice opportunities. It could be improved with greater/deeper opportunities for student verbalizations. |

1. **Master Evaluation Checklist for EngageNY/Eureka Math Lesson**

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| **Mathematics Lesson Plan Evaluation Checklist** |
| **Lesson:** | Order of Operations- EngageNY |
| **Grade Level:** | 6th grade |
| **Standard(s):** |  |
| **Objective(s):** | Student outcomes: Students evaluate numerical expressions. They recognize that in the absence of parentheses, exponents are evaluated first. |
| **Does the lesson include…** | **Rating 1 2 3**1 = missing2 = insufficient3 = sufficientN/A | **Notes and Suggestions for Revision/Adaptation** |
| **1. A clear teacher statement about lesson objectives?** | 3 | The teacher does state the objective, but not until the end of the whole-group discussion. The objective should be stated at the beginning of the lesson. |
| **2. Warm-up/review activities that assess student knowledge of critical pre-skills and activate relevant background knowledge?** | 2 | The opening includes a review of the problem set from the previous lesson, but no specific guidance on how to review/what to focus on or how to assess student readiness for the current lesson.The opening exercise asks students to independently evaluate an expression: 3 + 4 x 2; then tells teachers to have students re-check their work if they get an answer other than 11 or 14. This exercise could lead to confusion because students have not yet learned the order of operations. They do not yet know how to tackle this type of problem.To revise-An effective warm-up would provide review/practice with critical pre-skills. A relevant review could focus on the definition of ‘variable’ and identifying variables in expressions; as well as the definition of ‘expression’ and identifying expressions. These are critical pre-skills for success with the current lesson.The exercise is followed by a whole-class discussion which ends with a teacher statement about order of operations (we multiply and divide before we add and subtract). Explicitly explaining this rule to students would be a good way to start the model, but it doesn’t belong in the warm up. |
| **3. Modeling w/think aloud?**The lesson provides specific and sufficient guidelines for teacher modeling.\*The teacher shows and tells students how to meet a learning objective (i.e., provides clear, step-by-step demonstrations) using clear, concise, mathematically accurate language. | 1 | The teacher leads a whole group discussion in which they lead to a key point: *We now know that when we evaluate expressions, we must agree to use one set of rules so that everyone arrives at the same correct answer. These rules are based on doing the most powerful operations first**(exponents), then the less powerful ones (multiplication and division, going from left to right), and finally, the least powerful ones last (addition and subtraction, going from left to right).*However, the teacher does not MODEL how to apply the order of operations. He does not explicitly show and tell students how to apply the rule to evaluate expressions—he asks students to evaluate expressions independently from the very beginning.To revise, the teacher should provide at least 2 models in which he thinks aloud while applying the order of operations to carefully chosen example problems. In other words, the teacher should explicitly show and tell students how to solve at least 2 problems using the order of operations BEFORE asking them to engage in guided or independent practice. |
| **4. Segmentation of complex skills?**The lesson breaks a complex skill or strategy into smaller instructional units. | 2 | This lesson presents the order of operations in two chunks. First, students learn to evaluate exponents, then multiplication and division (left to right), and finally addition and subtraction (left to right). Next, the teacher explains any part of an expression in parentheses must be evaluated first.In effect, the order of operations is somewhat segmented within the lesson, but could be broken down further (by teaching one step at a time over a series of lessons, rather than teaching the entire order of operations in one lesson). |
| **5. Sufficient and appropriate instructional examples that are appropriately sequenced and scaffolded?** | 2 | The types of problems presented get more and more complex throughout the lessons and there are only ~2 practice problems of each kind. The lesson could be improved by focusing instructional examples and by providing several examples of each type with scaffolded support before introduction of a new type. |
| **6. Explicit instruction or review of key mathematics vocabulary terms using precise, student-friendly definitions?** | 1 | No vocab. instruction or practice provided.  |
| **7. Systematically faded supports?**The lesson promotes successful engagement by providing systematically scaffolded practice opportunities. Initial practice opportunities are heavily supported; then based on students’ response, supports are gradually faded to release responsibility and increase student independence.  | 1 | The teacher provides practice opportunities but support is not systematically faded. Students are expected to work independently from the beginning of the lesson. To revise, the teacher could use the Tell, Ask, Remind (TAR) framework for guided practice. To use the TAR framework, the teacher would provide at least 1-2 practice opportunities in which they lead students through solving step-by-step (i.e., tell), followed by 1-2 practice opportunities in which they prompt students to solve using questioning (i.e., ask), and finish with 1-2 practice opportunities in which they remind students of the rule and have them work more independently (i.e., remind). During guided practice, the teacher should closely monitor student responses to determine when students are ready for fading.  |
| **8. Many opportunities for students to respond with feedback AND opportunities for student verbalizations?** | 2 | The lesson does include question prompts and says that the teacher should circulate to monitor student work and provide feedback. It’s unclear from the lesson plan whether ALL students will get many opportunities to respond WITH feedback during the lesson. |
| **9. Purposeful practice?**The lesson provides practice opportunities that align with learning objectives. Practice is distributed and cumulative. Practice may provide opportunities to use concrete manipulatives and/or visual representations to model math skills/concepts. | 2 | For the most part, the students engage in practice aligned with the stated objectives (although the practice is not systematically faded).However, in Example 3, the practice diverts from focus on order of operations.The lesson provides a few problems using visual representations. VRs are included but not explicitly explained. Students need instruction in how to interpret VRs to use them effectively. |
| **10. Correction procedures or guidelines to address student misconceptions?** | 2 | The lesson says that teachers should reiterate the rule about nested parentheses if students make mistakes on exercises 6-7, but this is the only example of guidance for corrections or addressing misconceptions.  |
| **Final Evaluation:**Rating total (30 = highest possible score) | 17 | **Final Notes/Comments:**This lesson would really benefit from the addition of (1) clear teacher models; (2) a narrowing of the lesson focus and instructional objectives; and (3) scaffolded practice opportunities with feedback. |