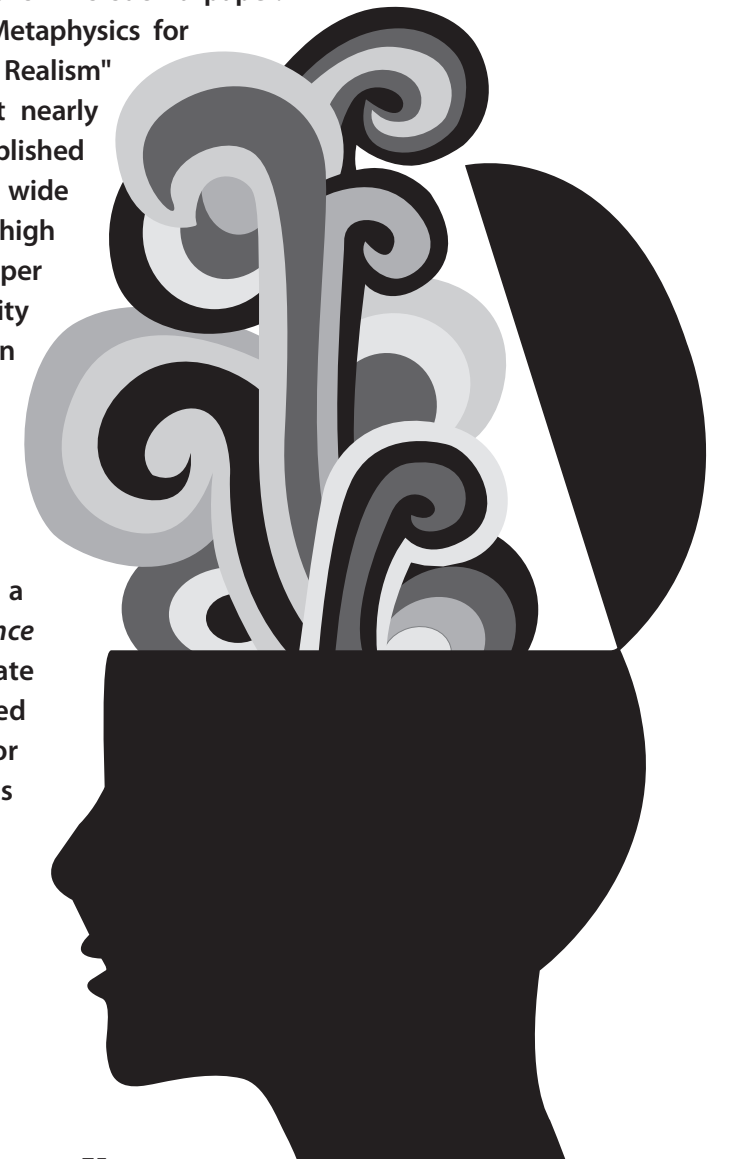


Special Interest Section

In very rare instances, completely at the discretion of the editors of *Stance*, a paper comes along that is given special consideration and a special place in the journal. "A Metaphysics for Mathematical and Structural Realism" is such a paper.

As an original submission, "A Metaphysics for Mathematical and Structural Realism" was highly impressive and met nearly all the criteria required to be published – all except an accessibility to a wide audience on which *Stance* places a high premium. The only way the paper could reach such an accessibility was to increase drastically in length. In a sense, *Stance* solicited further work from the author in hope of publishing a great undergraduate paper.

The Special Interest Section is a safeguard in that it allows *Stance* to recognize great undergraduate work that may not be confined to the Journal's requirements for submission while retaining its inherent quality.



A Metaphysics for Mathematical and Structural Realism

ABSTRACT: The goal of this paper is to preserve realism in both ontology and truth for the philosophy of mathematics and science. It begins by arguing that scientific realism can only be attained given mathematical realism due to the indispensable nature of the latter to the prior. Ultimately, the paper argues for a position combining both Ontic Structural Realism and Ante Rem Structuralism, or what the author refers to as Strong Ontic Structural Realism, which has the potential to reconcile realism for both science and mathematics. The paper goes on to claims that this theory does not succumb to the same traditional epistemological problems, which have damaged the credibility of its predecessors.



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Introduction and Context

The status of science and mathematics is perhaps one of the most important topics in the contemporary intellectual discourse, and hence one of the most fiercely debated. Since the initiation of the Scientific Revolution, western civilization has come under the governance of rationality, empiricism and reductionism - toward the general trend that epistemological authority has been increasingly surrendered to those involved in the activity called science, from

its historical base in philosophy or religion. This corresponds with the development, and increasing implementation, of rational instruments or mechanisms by which to induce order, predictability and control (administration, standardization, and bureaucracy). Scientists, and societies affected by the Enlightenment, have in turn become increasingly reliant on the activity called mathematics. Thus, the modern world is intimately connected to, and indeed rests upon, the mathematical and scientific realism. However, several alternative programs have significantly

challenged these underlying suppositions. The aim of this essay is to engage in this pertinent debate and to reconcile the objective nature of mathematical and scientific truth.

Why Mathematical and Scientific Realism

Why would one would desire to call him or herself a mathematical and scientific realist? Briefly, philosophical subjects usually divide along realist or antirealist lines. I define 'realism of x ' to be (i) the position that those objects which are in the 'domain of discourse of x ' are in fact ontologically significant and that these objects exist independently of the human mind and (ii) that statements made about those objects which are in the 'domain of discourse of x ' either hold true or false of those objects thereby establishing a truth value account for x . A second way to consider this is that a realist holds that the subject matter in question has a real ontological status and/or that ontological statements about the subject matter in question are not vacuous or fictitious. This is usually taken to mean that this subject matter is somehow "independent of anyone's beliefs, linguistic practices, conceptual schemes, and so on."¹ Antirealism can take many forms but antirealist claims usually rest on the notion that the subject matter is either fictitious, does not exist, or is dependent on someone's beliefs, linguistic practices, cultural constructs, and so on.

Mathematical realists hold that mathematical

objects are real and exist independently of the human mind, and that mathematical statements are about those objects and are therefore true or false.² The Quinean dictum "to be is to be the value of a [quantified] variable"³ is the relevant convention for the nature of mathematical objects and their relation to a mathematical statement.

Interestingly, it appears that the majority of working mathematicians are "working realists."⁴

A traditional scientific realist holds that scientific objects are real and exist independently of the human mind, and therefore scientific statements about those objects are true or false. Or in other words, because science operates on the basis of falsifiability, and the confirmation of individual results, we have good reason to take science and scientific statements "at face value."⁵ Clearly, mathematical or scientific antirealism jeopardizes the ability of mathematicians and scientists to be able to make truth claims, or claims to knowledge. Tentatively, I believe that we should accept realism in both mathematics (MR) and science (SR) on the intuitive grounds that this provides the simplest account for the success of these disciplines - that there being real sets, functions, quarks, electrons (etc.) offers the simplest account for the increasingly descriptive and applicable nature of these two activities. The antirealist will contend, of course, that there is nothing simple about this account. Importantly, the close relation of these two disciplines means that in some way, denying realism in one is bound to have ramifications in the other.⁶

It is my unrepentant assumption that scientific realism is a desirable end - that is I

1. Stanford Encyclopedia of Philosophy, "Realism," 4 Aug 2005, <<http://plato.stanford.edu/entries/realism/>> (29 November 2008).

2. More technically this might be considered realism in ontology and truth value, respectively.

3. Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology*, (Oxford University Press, 1997): 4. Stewart Shapiro, 7-8.

4. Stanford Encyclopedia of Philosophy, "Scientific Realism," 12 June 2002, <<http://plato.stanford.edu/entries/scientific-realism/>> (14 February 2009).

6. The strange connection between the sciences and mathematics has been well noted, see Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," in *Communications in Pure and Applied Mathematics* 13.1 (1960): 1-14.

ascribe to the notion that scientific discovery is a progressive march toward the objective nature of reality. This, I feel, is the optimum footing upon which to ground human knowledge. As such, I am sympathetic to *Ante Rem* Structuralism (ARS) and Ontic Structural Realism (OSR) as they seem like the best-bets to establish MR and SR, respectively. Furthermore, it is my tentative assumption that of the various positions arguing for MR and the various positions arguing for SR that the best chance to eliminate the long-standing *epistemological problem* in MR is found in a joint ARS and OSR position, or what I dub Strong Ontic Structural Realism (SOSR). The principle aim of this paper is to attempt to articulate a tentative metaphysical position that satisfies both MR and SR. However, some of the major problems confronting MR and SR will be discussed. I offer SOSR as a best bet for those seeking both MR and SR. The general assumptions of my argument are as follows:

- (0.0) SR \rightarrow Knowledge
- (0.1) OSR \wedge MR \rightarrow SR
- (0.2) ARS \rightarrow MR
- (0.3) OSR \wedge ARS \rightarrow SOSR

Scientific Structuralism

It is closely related to the mathematical structuralism. Scientific structuralism holds that scientific theories are to be characterized as a collection of models that share the same kind of structure,⁷ and that the objects talked about by a theory are positions in such models.⁸ The semantic view prevails in framing the contemporary scientific structuralism.⁹ This position “rejects the need for, and possibility of, correspondence rules and instead uses models, in the Tarskian sense, to provide an unmediated theory-world connection.”¹⁰ Its opposite, the syntactic view, holds “that a theory is an uninterpreted, or partially interpreted, axiom system plus correspondence rules, or co-ordinating definitions, that mediate so as to provide for the theory-world connection.”¹¹ Scientific structuralism differs strongly from the mathematical structuralism in that a scientific structuralism must realize clear distinctions between kinds of objects and particular objects, as well as between theoretical objects and their physical realization. In mathematics there is no such thing required, because there is no distinction that must be drawn between a theoretical object and a physical realizable object¹² – the reader should grasp that by Quine’s statement above, a mathematical object “exists” if it is bound by a quantifier in a sentence.

Scientific structuralism, or structural realism, has been offered as an account for SR. In the debate about SR, “arguably the two most compelling arguments around are the ‘no miracles’ argument, and the ‘pessimistic meta-induction.’” These two arguments pull in two different directions: naïve realism on the one hand and antirealism on the other. “In an attempt to break this impasse, and have ‘the best of both worlds’, John Worrall introduced *structural realism*.”¹³ That is, Epistemic Structural Realism (ESR) was originally offered as a sort-of pragmatic account for science, in the same vein as Instrumentalism, while simultaneously attempting to support the validity of realism in scientific truth.

ESR addresses these two problems by not making the success of science seem miraculous and not forcing us to commit to the claim that a theory’s structure describes the world - and by avoiding the force of pessimistic meta-induction, by not committing us to a belief in a theory’s description of the objects of the world - “according to the latter argument, we cannot commit ourselves to the belief in present theories since successful theories throughout the history of science were refuted or abandoned.”¹⁴ “ESR purports to identify the structural content of a theory in such a way as to ensure cumulative continuity in that kind of

content.”¹⁵ Hence, ESR is concerned with the preservation of scientific continuity which has been disputed by such thinkers as Kuhn¹⁶ and is motivated by the notion that while scientific paradigms have shifted radically, certain mathematical equations seem to have remained consistent. Scientific structuralism responds to discontinuity by asserting that certain structural features of differing scientific theories remain stable, even in the face of radically revised scientific ontology.¹⁷ Essentially, ESR asks us to commit only to the mathematical content of scientific theories. Thus, ESR admits that our actual knowledge of *things-in-themselves* is limited at best. What we can know, are the structural features of whatever there is in reality and that those objects have structural content. Hence, realism in scientific truth is preserved.

OSR is a more radical thesis. OSR denies the epistemic limitations of ESR by asserting a revisionist metaphysical claim: essentially, that our traditional ontological category of object-hood is incorrect, that only structures exist in the world.¹⁸ Objects, are merely conventions to discuss or conceptualize things. This is taken to mean that “‘Structures have ontological primacy over objects’ and this ‘either means [1] that structures are all that exist or [2] that entities are dependent for their own existence on the existence of structures.’”¹⁹ This position is closely related to the mathematical

7. The semantic view is the view that a theory is a collection of models (model, in a model-theoretic sense). The syntactic view demands that scientific theories provide some additional non-structural or non-mathematical information to describe theories (theories as sets of natural language sentences).

8. Katherine Brading and Elaine Landry (1), “Scientific Structuralism: Presentation and Representation,” in *Philosophy of Science* 73.5 (2006): 573.

9. Katherine Brading and Elaine Landry (2), “A Minimal Construal of Scientific Structuralism,” 29 January 2005, <<http://philsci-archiv.pitt.edu/archive/00002181>> (accessed 20 November 2008): 5.

10. Katherine Brading and Elaine Landry (2), 7.

11. Katherine Brading and Elaine Landry (2), 6.

12. Katherine Brading and Elaine Landry (1), 3.

13. James Ladyman, “What is Structural Realism?” in *Stud. Hist. Phil. Sci.* 29.3 (1998): 409.

14. Aharon Kantorovich, “Particles vs. structures: Weak ontic structuralism,” 2 December 2006, <<http://philsci-archiv.pitt.edu/archive/00003068/>> (accessed 16 February 2009): 2.

15. Juha Saatsi, “Whence Ontological Structural Realism,” 11 May 2008, <<http://philsci-archiv.pitt.edu/archive/00004016/>> (accessed 21 November 2008): 2.

16. See Thomas Kuhn, *The Structure of Scientific Revolutions*, (Chicago: University of Chicago Press, 1962).

17. Katherine Brading and Elaine Landry (2), 21.

18. Anjan Chakravartty, “The Structuralist Conception of Objects,” in *Philosophy of Science* 70 (2003): 867-68.

19. Aharon Kantorovich, 17.

structuralist's conception of mathematical objects. If only structures exist, then we are justified in taking the structuralist conception of scientific statements at face-value. OSR is significantly motivated by work in quantum mechanics²⁰ where the status of individuality and object-hood are underdetermined.²¹ For example, Leibniz's Principle of the Identity of Indiscernibles²² appears to suggest that many subatomic particles, which are understood to be "individual objects" are in fact the same object. It should be noted that this particular problem, along with the ontological status of the wave function, has served as a traditional point of division between realists and antirealists.

To give a realist account for science, it must be demonstrated that scientific theories have in some way been characterized by a shared structure or continuity. Scientific structuralism provides grounds to do that, however ESR appears to only shakily satisfy the first criterion of the definition for realism offered in section I, because only the structural content of objects is acquired. The *epistemological problem* for how exactly abstract mathematical structures in any way are related to physical objects still lingers against this ESR "weak commitment."

Adopting an OSR stance enables the realist to firmly accept both criterion for SR and therefore to take scientific theories at both ontological and epistemic face-value, though the trade-off requires a rewriting of our ontological assumptions. Hence, OSR carries a heavy metaphysical

commitment, or rather uncommitment. However, work in quantum mechanics, seems to indicate that particles at the quantum level seem to violate our principle understanding of what qualifies something as an object, at least in the classical sense. For the antirealist this is merely an artifact of fallible human science. For the realist, the classical conception of physical objects must be incomplete under this picture. Kantorovich clarifies, "individuals can be viewed as 'different representations of the same structure' (ibid). This statement can be understood most clearly when the structure is a symmetry group."²³

As scientists have become more reliant on mathematics to describe the physical features of the world and as mathematical activity itself appears to be heavily characterized by structuralist tendencies,²⁴ the antirealist must explain why this strange relationship proves so fruitful to science and technology, and to those insisting on a traditional metaphysical framework I offer a second challenge: Do we have any good reasons to remain steadfast in our object-based ontology. Why must objects take primacy in our ontology?

Mathematical Structuralism

The primary alternative to MR can be found in mathematical constructivism, which is a family of related but distinct forms of antirealism. Two variants stand out: (i) Social

Constructivism, maintains that mathematics is primarily a work of human social conventions and (ii) Intuitionism, a finitist mathematical philosophy, asserts that mathematical objects must be finitely constructed, or step by step, because mathematical objects are mental constructions in the mind of the mathematician. I find it difficult to explain, if mathematics is a human construct, how exactly SR can be preserved for those mathematical antirealists leaning toward SR, given the immense connection between the two as noted above.

Social Constructivism faces the challenge that if mathematical concepts are social conventions, then why is it that certain mathematical concepts have held true throughout the ages and across cultures? Intuitively, $1+1=2$ seems universally valid even though our philosophy of mathematics, or understanding and explanation of mathematics, has seen dramatic change. An oft cited counter-example to MR is found in the 18th century conception that Euclidean geometry was to be considered the *a priori* description of space itself. Clearly this idea was dismissed with the development of hyperbolic and non-standard geometries in the early 19th century. The social constructivist takes this as evidence that mathematics are contingent, that the axioms and assumptions upon which the human mathematical activity rest, as well as the intention of those axiomatic frameworks, are subject to change depending on the cultural and social contexts of an

era. The MR would respond that Euclidean geometry is a real mathematical structure, as is hyperbolic and the non-standard geometries. And, while Euclidean geometry may not be the most fruitful mathematical system to describe space-time, to assert that this in some way violates the absolute nature of mathematics is to confuse applicability of mathematics with mathematics itself. Certainly mathematicians are influenced by the social circumstances of their age, they are human beings after all, but this does not negate that the mathematical enterprise ultimately comprises a description of some objective reality.

Intuitionism faces problems of its own. If mathematical objects are merely mental constructions, how then can we say that the mathematics of one person is the same as another's?²⁵ In addition to this problem, Intuitionism rejects much of that which is classically provable on the grounds that proof requires existence. This, and the additional Intuitionist requirement that both the Law of the Excluded Middle ($P \vee \neg P$)²⁶ and the Law of Double Negation ($\neg\neg P \rightarrow P$)²⁷ are not necessarily universal rules and therefore not valid in proving theorems. Notably, few working mathematicians have adopted Intuitionism on philosophical grounds.²⁸ Of related importance, there is general agreement among the philosophers of mathematics that philosophical positions should "give an account of mathematics as it is practiced, not to recommend sweeping reform."²⁹

There is a second group of antirealist positions

20. See Dean Rickles and Steven French, "Quantum Gravity Meets Structuralism: Interweaving Relations in the Foundations of Physics," in *The Structural Foundations of Quantum Gravity*, (Oxford: Oxford University Press, 2006): 1-39.

21. Décio Krause, "Remarks on Quantum Ontology," in *Synthese* 125 (2000): 162.

22. Explained in further detail below.

23. Aharon Kantorovich, "Particles vs. structures: Weak ontic structuralism," 2 December 2006, <<http://philsci-archive.pitt.edu/archive/00003068/>> (accessed 16 February 2009): 11.

24. A great example of this can be found in the development of Category Theory which takes undefined "objects" in its ontology and attempts to define the more or less structural relations of those objects - viewing objects as placeholders.

25. Penelope Maddy, *Realism in Mathematics*, (Oxford: Clarendon Press, 1990): 23.

26. The Law of the Excluded Middle essentially says either something is true or it is false (but not both or neither).

27. The Law of Double Negation essentially reads 'it is false that it is false that P' is the same as P.

28. A significant number of classical mathematicians do work in constructive mathematics because of its important applications in the development of Strong AI and computer science.

29. Penelope Maddy, 23.

found in (iii) nominalism *about* mathematics, which is part of the larger nominalism which denies the existence of abstract objects.³⁰ This is a complicated view, which essentially denies that abstract mathematical entities exist, a position strongly motivated by naturalism which is the philosophical position that the natural laws and/or the scientific metaphysics is alone sufficient to explain reality. To date, “nominalist mathematics” have failed to generate even a small fraction of what is classically provable. Regarding the naturalist motivation, it appears that at least some universals are required to adequately explain scientific theories.³¹ The last major antirealist contender is Formalism (iv), which holds that mathematics is a meaningless activity characterized by the manipulation of strings of symbols.³² Traditionally, Formalism has also been a finitist philosophy of mathematics but with very different aims than Intuitionism. It was the express goal of Hilbert’s Program to be able to generate a consistent set of axioms from which every possible classical mathematical theorem could be derived procedurally as a means by which to secure the absolute certainty of mathematical truth. However, this activity was more or less halted by Kurt Gödel, a Platonist, who proved with his famed Incompleteness Theorems that no such

axiomatic framework was possible.³³

The Zermelo Ordinals

0	\emptyset	0
1	$\{\emptyset\}$	{0}
2	$\{\{\emptyset\}\}$	{1}
3	$\{\{\{\emptyset\}\}\}$	{2}
.		
.		
.		$1 \notin 3$

The Von Neumann Ordinals

0	\emptyset	0
1	$\{\emptyset\}$	{0}
2	$\{\emptyset, \{\emptyset\}\}$	{0, 1}
3	$\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$	{0, 1, 2}
.		
.		
.		$1 \in 3$

Traditionally, the mathematical realist has held a Platonist conception of mathematics - that there is an abstract independent mathematical reality that “contains” the actual objects talked about by mathematical statements, usually intended to be sets or numbers, and that these objects act as Platonic Forms in regards to

the physical universe. This, of course seems metaphysically problematic. If there are two independent realities, how do they relate? This is the *epistemological problem*. Another famous problem confronting the mathematical realist’s epistemology was raised by Benacerraf: given that a mathematical realist asserts that the natural number line is real, and that we can define the natural number line in an infinite number of ways, how can the mathematical realist instantiate which of these formulations the natural number line is?³⁴

In the prior case the number one stands independent of the number three. In the latter case the number one is understood to be “contained in” the number three. This means that the relevant criterion of individuation, namely, Leibniz’s Principle of the Identity of Indiscernibles, does not hold.³⁵

$$\forall F(Fx \leftrightarrow Fy) \rightarrow x=y^{36}$$

Identity of Indiscernibles (PI)³⁷

Essentially this antirealist claim contends that there are two principle problems with the traditional Platonist conception of the natural number line: (i) If the natural number line is a universal then, it should be the case that each natural number system should be identical using (PI), and (ii) if the mathematical realist asserts that mathematical statements are true in

virtue of the fact that they name an ontologically significant object, then they should be able to pick out which natural number line they are speaking about. Given that there are no particular reasons why one should be inclined to talk about one natural number system over another, Benacerraf concludes “that numbers are not objects, against realism in ontology.”³⁸ If this is the case, then it seems difficult to accept that most of the mathematical enterprise, which is reliant on the natural number line, conforms to the mathematical realist’s vision which is a criterion for full-blown MR by section I.

Shapiro maintains that ARS enables us to answer this question, and others, thereby preserving the realist position. In the above case, each defined natural number system is a particular instance of an abstract natural-number structure.³⁹ That is to say, the two natural number systems above are isomorphic to each other and thereby demonstrate the existence of an abstract structure that they exemplify - that it is wrong to range (PI) over the individual numbers because there are no natural numbers as particular objects - that is, as existing things whose “essence” or “nature” can be individuated independently of the role they play in a structured system of a given kind.⁴⁰ Thus, (PI) applies to the structural content of the two systems and confirms that they are identical because each “number” in one system lines up

30. There is a subtle difference when we speak of abstract versus concrete mathematical objects as opposed to “everyday objects” – nominalism in mathematics, asserts that only particular mathematical objects exist, whereas the Platonist in mathematics maintains that there are general mathematical objects and that these objects exist outside of space-time. See Øystein Linnebo, “The Nature of Mathematical Objects,” in *Proof and Other Dilemmas: Mathematics and Philosophy*, eds. B. Gold and R. Simons (Washington: Mathematical Association of America, 2008): 205.

31. See Bernard Linsky and Edward N. Zalta, “Naturalized Platonism vs. Platonized Naturalism.” in *The Journal of Philosophy* XCII.10 (1995):525-555.

32. Penelope Maddy, 23-24.

33. Stanford Encyclopedia of Philosophy, "Hilbert’s Program," 31 July 2003, <<http://plato.stanford.edu/entries/hilbert-program/#2>> (accessed 17 February 2009).

34. Stewart Shapiro, 5.

35. Katherine Brading and Elaine Landry (1), 572.

36. This essentially says two things are identical when all the properties that are true of one thing are the same as all the properties that are true of the other (and vice-versa).

37. Stanford Encyclopedia of Philosophy, "The Identity of Indiscernibles," 31 July 1996, <<http://plato.stanford.edu/entries/identity-indiscernible/>> (accessed 18 November 2008).

38. Stewart Shapiro, 5.

39. Stewart Shapiro, 5-6.

40. Katherine Brading and Elaine Landry (2), 572.

in one-to-one correspondence with a “number” in the second – that the relevant criterion for identity is isomorphism⁴¹ – essentially that there exists a one-to-one “structure preserving”⁴² map between two structures that preserves relations and objects in those relations.

Structuralist philosophies of mathematics hold that mathematics is primarily the free exploration of structures. A mathematical structure is a set with defined relations attached to that set.⁴³ A common feature of structuralism is that mathematical objects⁴⁴ are regarded as places or placeholders within a structure. Relations⁴⁵ link these placeholders such that structuralist objects, properly conceived, are defined by their associated relations within a structure.⁴⁶ The inner content or intrinsic properties of objects within a structure cannot be analyzed. To analyze the inner content of an object, one must fix that object as the domain of discourse making it the new structure under study. This process can be repeated indefinitely “downward.” It is understood that to avoid such an infinite regress, there is usually a background ontology selected (which is understood to be structurally irreducible) or fixed to a particular structure theory,⁴⁷ which Shapiro maintains is a deciding factor for adopting *ante rem* structuralism over categorical *in re* structuralism.⁴⁸

There are three predominant types of mathematical structuralism. *In re* or *eliminative* structuralism is friendly to nominalist treatments in mathematics. *In re* structuralism contends that mathematical structures exist only in virtue of actual instanced mathematical systems and that structures are ontologically reducible (hence its close relation to nominalism in mathematics).⁴⁹ Shapiro asserts that ARS is friendly to Platonist treatments of mathematics. ARS holds that structures satisfy the notion of abstract universals. A particular mathematical theory is an instance or system of that abstract universal. These abstract structures are understood to exist regardless of whether or not there exists a system exemplifying that structure.⁵⁰ Both ARS and *in re* structuralism can utilize the set theoretic background ontology with differing implications, the alternative is categorical structuralism which contends that category theory can serve as a background ontology for mathematics, and as a theory to describe the nature of structures in general. Categorical structuralism is usually related to *in re* structuralism but it may also support ARS (with the addition of a background ontology such as set theory). For the *in re* structuralist any background ontology may serve as the domain of discourse⁵¹ and, true to its name, no special commitment must be made by the eliminative

structuralist. It is the task of ARS to develop a structure theory to formally model their respective positions⁵² - a theory “strong enough to encompass [the behavior] of all structures.”⁵³ A structure theory is a collection of axioms, or statements, which describe how structures behave. Category theory, as mentioned earlier, does not attempt to say “what is being structured” only that this is how “something would behave” if it were plugged into the language of category theory. Shapiro outlines an axiom highly relevant to our discussion, the *Coherence Axiom*: “A structure is characterized if the axioms are coherent”⁵⁴ - If P is a coherent sentence in a second-order language, then there is a structure that satisfies (entails or “makes true”) P.⁵⁵

If we are to fulfill the mathematical realist’s mission we must satisfy both criterion outlined in section I, thus it does not suffice to eliminate background ontology – as that is the very thing required to preserve MR. ARS is motivated by three major concerns: (i) addressing the principle challenges to MR (ii) preserving the default position of Platonist realism in mathematics for working mathematicians⁵⁶ (iii) characterizing the actual behavior of mathematical activity.⁵⁷ Clearly, at the present time mathematical structuralism seems like the best-bet for MR, and of its variants, ARS addresses all three concerns whereas the *in re* structuralist appears to have difficulty with (ii).

Strong Ontic Structural Realism

I have hopefully demonstrated that ESR is not sufficient for a full-blown SR under the requirements laid down in section I, that OSR alone can accomplish this task and that ARS is the best-bet for MR. However, there are several problems confronting the combination of these two into a united MR and SR position. The *epistemological problem* looms large asking, if we are concrete physical creatures, how do we account for our abstract mathematical knowledge? The distinction between abstract and concrete objects is of significance to contemporary philosophy⁵⁸ - as long as a distinction is made between “nonphysical” and “physical” kinds. Some account for how these interact, or are related, is required. Thus, I approach this problem as a fundamentally metaphysical dilemma. I seek to offer a tentative characterization of SOSR which might aid in the resolution of this problem while simultaneously supporting both MR and SR. This position is partially motivated by Tegmark’s recently defended Mathematical Universe Hypothesis (MUH). In short, Tegmark argues that there is a physical correlate for every mathematical structure and that ultimately “our successful theories are not mathematics approximating physics, but mathematics approximating mathematics.”⁵⁹

41. Stewart Shapiro, 93.

42. Stewart Shapiro, 91.

43. Model theory is the premiere formal tool used to investigate differences between mathematical structures.

44. Such as numbers, sets, groups, etc.

45. Such as ‘+’, ‘<’, etc.

46. Stewart Shapiro, 82.

47. Stewart Shapiro, 82, 86.

48. Stewart Shapiro, 87.

49. Stewart Shapiro, 9, 86.

50. Stewart Shapiro, 84.

51. Stewart Shapiro, 87.

52. Stewart Shapiro, 90.

53. Penelope Maddy, 173-174.

54. Stewart Shapiro, 133.

55. Stewart Shapiro, 95.

56. Øystein Linnebo, “Platonism about mathematics,” forthcoming in Stanford Encyclopedia of Philosophy, 3 September 2008, <<http://seis.bris.ac.uk/~plxol/SEP-platonism.pdf>> (accessed 16 February 2009).

57. Jessica Carter, “Structuralism as a philosophy of mathematical practice,” in *Synthese* 163.1 (2008): 119.

58. Stanford Encyclopedia of Philosophy, “Abstract Objects,” 19 July 2001, <<http://plato.stanford.edu/entries/abstract-objects/>> (accessed 16 February 2009).

59. Mzx Tegmark, “The Mathematical Universe,” in *Foundations of Physics* 38.2 (2008): 105.

(ESR) If we admit that in some way physical structures are associated with mathematical structures

(OSR) And, if we admit that everything that is physical is structural such that individual objects do not exist or are merely places in a structure and that reality is physical

(SR) And, if we admit that scientific knowledge is primarily the investigation of the features of these physical structures

(ARS) And, if we admit that mathematics is primarily the free exploration of abstract structures and that these structures, in some way, act as universals

(MR) And, if we admit that mathematical structures are real, abstract and “independent” of the physical universe in the sense that mathematical structures are not reliant on the physical

(Arg1) Then, the simplest explanation for how mathematics corresponds to the physical universe is that the physical universe is itself an abstract mathematical structure⁶⁰

(Arg2) And, under the assumption that fewer ontological kinds are preferable to the multiple if those fewer kinds are sufficient to describe reality then, it follows only mathematical structures exist.

I will ask the reader to indulge me for a moment and join me in contemplating reality under this picture. First, as mathematics appears to be unified so would a mathematical reality be unified. How we perceive this reality likely divides the world into sense-perception and “actuality,” as the mechanism by which

we view the world may be illusory – clearly we do not see “little ones” floating around. In order to help conceptualize this picture I would like to first draw a distinction between formal languages and abstract mathematical structures and second, to discuss a physical thing as we intuitively grasp it, and a physical thing *in-and-of-itself*.

As per *Coherence Axiom* any consistent and coherent sentence in a second-order language has a corresponding abstract mathematical structure which satisfies it. The sentence “ $1+1=2$ ” has a corresponding mathematical structure that is characterized by the model theoretic symbol $\{ |N|, +, 0, <, x \}$ ⁶¹ which satisfies it. The symbols ‘1’ and ‘2’ are describing what is equivalent to the first two places of the natural number line which can likewise be symbolized |, ||, |||, ||||, ... which is itself characterized by the axioms of ZFC set theory.

When someone “suggests that some mathematical objects can resemble or ‘approximate’ physical objects like pieces of rope, they clearly do not mean that some mathematical objects are solid, flexible and flammable. You cannot twist or burn a number, even approximately.”⁶² One might inquire “how can the number one have a physical counterpart?” The traditional Platonist response is that individual numbers act as universals, such that each singular physical thing participates in the abstract universal. From the SOSR view, mathematical structures act as universals for individual physical things. When we talk about physical objects – say the piece of paper you are reading – we tend to take the naïve realist view and associate what we see, feel, hear,

taste, and smell as being the physical objects *in-themselves* such that we say “this piece of paper is white, smooth, crinkles when I bend the corner, etc.” Now, the conception of a physical object devoid of those sensations seems to stand in rebellion to our common sense – we do not like the notion that the “physicality” of the paper has nothing to do with its whiteness, its feeling of texture, the crinkle of its edge, etc. However, the status of physicality and what it means to be a physical object is itself a subject of much debate.⁶³ Properly understood, OSR suggests that all there is to physical things is that they are structures or that they stand within a structure – that the naïve realist conception of physical things is illusory. Opponents of OSR have challenged that such a structural view essentially collapses the distinction between physical and mathematical things. I am arguing that such a distinction is faulty in the first place. SOSR suggests that physical and mathematical things are one in the same. A physical entity is physical because it is a position in a mathematical structure.

Under a SOSR scheme our physical universe can be seen as being a finite subset out of an infinite mathematical reality. As for the apparent physical/abstract and concrete/universal oppositions, under SOSR such distinctions are trivial. A mathematical structure is both physical and abstract. Each concrete physical thing is an exemplification of that structure – a place in a universal. The motivations for finitism usually lie along the premise that natural physical reality is finite, and that such a reality is all that there is. Obviously, under SOSR finitism is an absurd notion. Properly understood, SOSR says

that “*abstraca*” is merely linguistic shorthand for the collection of mathematical structures that we have not yet found a physical correlate to.

On the *epistemological problem*: if the world is structures then, the mathematician accounts for mathematical knowledge acquisition empirically. The mathematician develops a language sufficient to talk about all structures in the world, and which can consistently talk about the “most” abstract mathematical structures that have not been empirically observed. The languages which accomplish this the best, are the languages which have historically been selected out over those that do not – a clear example is found in the refutation of Cantor’s naïve set theory⁶⁴ for ZFC set theory, the debate over the status of set theory as a foundational language given the suggestion that category theory may serve as a superior language, and the general acceptance of ZFC set theory over Intuitionist set theories, as previously mentioned. This provides a resolution to the *epistemological problem* in three ways: (i) the distinction between concrete and abstract is trivial, (ii) “mathematical intuition” can be replaced with mathematical empiricism and (iii) SOSR grounds the development of mathematical languages in an evolutionary framework.

Lastly, I offer a tentative approach toward the resolution of the *problem of universals* working from a group-theoretic analogy. Working from the position that all universals are mathematical structures, let us imagine two people looking at a single cardboard box. One person views this box from the side, the other person from a top-down bird’s eye view perspective. The side of the box is colored blue; the top of the box is

60. Max Tegmark, 101.

61. Kees Doets, “Basic Notions” in *Basic Model Theory*, (Stanford: CSLI Publications, 1996), 1.

62. Stewart Shapiro, 251.

63. See Henry P. Stapp, “Physicalism Versus Quantum Mechanics,” arXiv.org/ (11 May 2008), <<http://arxiv.org/abs/0803.1625>> (21 February 2009): 1-20.

64. Which allowed paradoxical sets to be formed.

colored red. Now, let us imagine that we were to ask these two people if they saw the same object. The person viewing the box from the side would say, “I have seen a blue square” and the person who viewed the paper from the top-down perspective would say “I have seen a red square.” Quite naturally, we might presume that these two people wouldn’t agree that they had seen the same object. Clearly, two differently colored squares cannot be the same object as it appears that the properties (namely blueness and redness) of the two independent objects are not identical. In a related thought experiment we take the two observers standing at the same position, though at two different times, and rotate the box between those times such that for the first person the “square appears red” and for the second person “the square appears blue.” Again these two observers might be inclined to argue that they had seen two different objects. Now we, as omniscient observers in this thought experiment, recognize that objects can undergo rotation, such that for two different observers, or from two different perspectives, the same object can appear as many. Groups, specifically symmetry groups, capture this notion and are an indispensable and fundamental tool in the contemporary physics.

I will extend this rough idea a bit further: if physical objects are positions in universal mathematical structures then, these concrete physical things may appear as individual, separate entities while actually being “sides,” or at least places in, a single mathematical structure, or possibly “rotations” of a higher

dimensional mathematical structure that then serves as a universal, giving the illusion of enduring over time. Some related evidence toward this can be found in the theory that our human visual perception of three dimensions is actually captured in a two-dimensional projection surface and/or the theory that the traditional conception of a four dimensional space-time may actually be reducible to two dimensions as per the Holographic Principle - that our experience of three dimensions or three dimensions plus time, and the objects within them, may be somewhat of an illusion and their actual nature may be radically different from how we perceive them.

Closing Remarks

There are a number of significant problems confronting this position. Obviously the status of OSR and ARS is underdetermined and the prevailing philosophical winds could possibly swing toward the antirealist position. There is also the fact that the predominant contemporary metaphysics is framed in nominalism, naturalism and physical reductionism,⁶⁶ so SOSR, and related metaphysics, are likely to meet great resistance and to be considered greatly revisionist. Furthermore, SOSR requires a great deal of clarification before any formalization can be undertaken toward a full and extended metaphysical position. And until a formal account is developed to demonstrate how exactly such a universal mathematical structure might be characterized that it would

resolve the *problem of universals*, such a claim is clearly only speculation.

It is my view that just as science and philosophy stood at a crossroads facing the perplexing contradictions between the long-held Newtonian world-view and the startling new quantum mechanical paradigm science and philosophy today are likewise undergoing significant changes. Buzzwords like “emergentism,⁶⁷ structuralism and consciousness” represent the striking fact that much of what was considered improper to the domain of scientific activity has actually been

incorporated into the highest levels of scientific activity over the last fifty years. The relevance of mathematics, and its strange connection, to all of these activities inclines me to believe that a fundamental metaphysical revision is required. Thus, I offer SOSR as stepping-stone in that direction. Clearly, substantial is required to flesh out this position – however, given its possibility toward resolving a number of classical philosophical problems, it is an area I hope others will be inclined to find fruitful. ♦

65. Essentially, that n dimensions can be captured in n-1 dimensions. See Leonard Susskind, “The world as a hologram,” in *J. Math Phys.* 36.11 (1995): 6377-6396.

66. Thomas B. Fowler, “Reductionism, Naturalism, and Nominalism: the “Unholy Trinity” and its explanation in *Zubiri’s Philosophy*,” in *The Xavier Zubiri Review* 9 (2007): 69.

67. The position that reducing a system to its constituent parts does not give one the full mechanics of that system.