Science: A Greatest Integer Function–A Punctuated, Cumulative Approach to the Inquisitive Nature of Science

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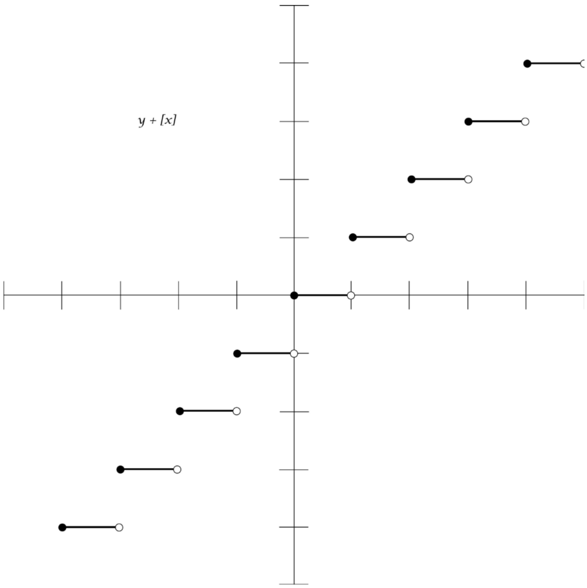
Abstract: Thomas Kuhn argues that scientific advancements sometimes involve paradigm shifts between incommunsurable theories, thoughts, and concepts. I argue that the phenomenon Kuhn is attempting to describe is better explained as akin to a greatest integer function of punctuated equilibrium. I conclude that Kuhn is mistaken in thinking that science is an actively vigorous, cumulative discipline.

I. Preface

Consider a greatest integer function or step function as in Diagram 1.0. A greatest integer function is a special type of discontinuous function whose graph is a series of line segments. It is a cumulative distribution function of a random variable and jumps from one value to the next, therefore resembling a series of steps. One endpoint in each step is closed (black dot) to indicate that the point is a part of the graph and the other endpoint is open (open circle) to indicate the point is not a part of the graph.[1]

[1] J. Stewart, *Calculus 7E Early Transcendentals* (Boston: Brooks-Cole, 2010): 29.

Diagram 1.0 Greatest Integer Function



Now, consider the evolutionary biological theory of punctuated equilibrium, as depicted pictorially via Diagram 1.1. Eldredge and Gould's theory of punctuated equilibrium articulates long periods of apparent stasis interrupted by relatively brief periods of sudden change, as demonstrated below.

Diagram 1.1 Punctuated Equilibrium Graph

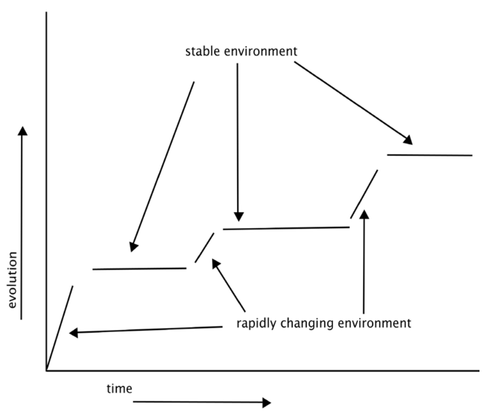


Diagram 1.0 and Diagram 1.1 share striking resemblances; both graphs involve a discontinuity at certain points, depict a cumulative distribution of a random variable, and both graphs resemble a series of steps. From a holistic viewpoint, the steps these two graphs depict are merely parts of a greater staircase called *progress*. However, it is each individual step which promotes transformation, revolution, and improvement.

II. Introduction

Science is dynamic. It is a unique discipline which centers on the concept of revision. It recognizes the basic uncertainty of human knowledge and utilizes that uncertainty to establish its inquisitive nature. Science is an actively vigorous discipline.

What is the nature of scientific advancement and progression? Many discussions have occurred regarding the nature of science, and much work has been done to investigate various scientific methods and diverse modes of scientific enquiry by several philosophers of science. Most notably in this regard is Thomas Kuhn. Kuhn's book, *The Structure of Scientific Revolutions*, sparked most contemporary responses to these questions, as it is unquestionably the most influential work in philosophy of science during the last fifty years.[1, 2]

In this essay, I will attempt to cast doubt on Kuhn's general argument that the development of science occurs via juxtaposed paradigm shifts in incommensurable theory, thought, and concepts. I will then try to respond to the question initially posed (i.e. What is the nature of scientific advancement and progression?) by arguing that the simplest way to answer this question is to liken science to a greater integer function of punctuated equilibrium. Thus, I will attempt to respond to this question by reinstating the long-established notion that science is not a diminishing discipline, but is rather a cumulative discipline.

Kuhn challenged the prevailing notion of the nature of science as cumulative and progressive, arguing instead that science evolves through revolutionary changes in which one theory or "paradigm" is replaced by a radically different one.

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[2] S. Okasha, *Philosophy of Science: A Very Short Introduction* (New York: Oxford University Press, 2002): 77.

The replacement of existing theories with radically different ones did not merely involve a simple matter or theory substitution, but it also involved a paradigmatic shift in concept meanings from previous theories.

Kuhn distinguishes between normal and revolutionary scientific development by arguing that most successful scientific research results in change of normal science, and "its nature is well captured by a standard image: normal science is what produces the bricks that scientific research is forever adding to the growing stockpile of scientific knowledge." However, Kuhn adds, "Revolutionary changes are different and far more problematic. They involve discoveries that cannot be accommodated within the concepts in use before they were made. In order to make or assimilate such a discovery one must alter the way one thinks about and describes some range of natural phenomena."[3] He therefore concludes that since "referential changes of this sort accompany change of law or theory, scientific development cannot be quite cumulative."[4] Normal science is practiced within a certain paradigm, which provides the scientist with puzzles to solve. Once a large amount of anomalies has accumulated, or a particularly troublesome anomaly that cannot be ignored is encountered (due to the insufficient or inadequate current paradigm within which scientists are working), a new paradigm may be formulated, encompassing all of the anomalies that existed in the previous paradigm. The newly formulated paradigm is thus adopted, thereby resulting in the abandonment of the previous paradigm. With this new paradigm, however, not only comes a novel theory, but also comes a novel way of interpreting older concepts and definitions. Thus, a paradigmatic shift in theory, thought, and concepts occurs, marking the occurrence of, as Kuhn calls it, a revolution.

Visualizing Kuhn's argument, we can liken scientific development and novel discoveries as a series of bubbles juxtaposed against each other.

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[3] Thomas Kuhn, *The Structure of Scientific Revolutions,* 2nd ed., rev. (Chicago: University of Chicago Press, 1969): 7-8. The book was first published in 1962.
[4] Ibid., 8.

Diagram 2.0 Kuhn's Account for Paradigms and Anomalies

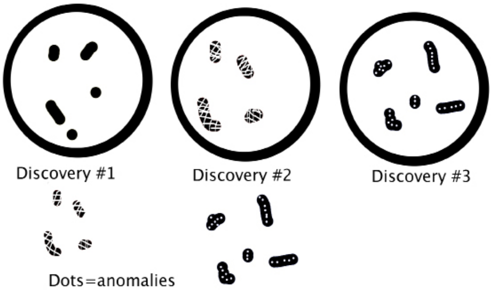

Diagram 2.0 is a pictorial depiction of Kuhn's argument that scientific development and novel scientific discoveries are not cumulative, but rather that each revolutionary change is "somehow holistic."[5] Each discovery (Discovery #1, Discovery #2, and Discovery #3, respectively) was advanced through an accumulation of encountered anomalies. As scientists studied each anomaly, they were eventually able to develop a new paradigm, which resolved the anomalies encountered in the previous paradigm. This enabled and encouraged them to abandon the previous paradigm and the concepts and meanings each paradigm housed. According to Kuhn, developments in science are therefore neither cumulative nor uniform but instead occur in alternating periods of normal and revolutionary science.

III. Critique of Kuhn

Kuhn's notion of scientific development through independent paradigmatic shifts in theory, thought, and concepts is problematic in several ways. First, Kuhn's concept of paradigms is unrealistic. Second, Kuhn's position on paradigms being incommensurable is too radical. Finally, Kuhn's use of an evolutionary metaphor to explain science's pursuit of truth is troublesome. From a deeper understanding of each critique, it will become clear that Kuhn's overall view of the nature of science and its development through incommensurable paradigmatic shifts in theory, thought, and concepts is problematic.

[5] Ibid.,19.

First, Kuhn's concept of paradigm is unrealistic. Kuhn argues that the development and transformation of science is catalyzed by formulations of novel paradigms that articulate and encompass the "puzzles" the existing paradigm encounters. Thus, normal science operates under a given paradigm, which simultaneously provides the scientist with both puzzles to solve and tools to solve each given puzzle. Once too many of the pieces of the puzzle prove incompatible with each other and the given theory, a new paradigm is adopted and the former paradigm is abandoned, along with its concept meanings. Kuhn maintains that both paradigms are incomparable, incompatible, and thus, incommensurable.

Kuhn's concept of paradigm is unrealistic because he fails to consider the dynamic nature of science. Science is founded upon revision and is always changing; scientific revisions, and thus scientific advances, are made much more frequently than Kuhn asserts. While it may be true that scientific revolutions are rare, revisions in theory, thought, and concepts often occur—although they may not be nearly as dramatic as Kuhn argues. These revisions in scientific theory, thought, and concepts occur during what Kuhn would call, "normal science." To ignore these revisions in science—however minute they may be—is to ignore the overall dynamic nature of science. Science is not a static discipline.

Second, Kuhn's position on the incommensurability of paradigms is too radical. To assert that two rival theories share neither common meanings nor observations is far too extreme. Through the practice of constant revision, science effectively builds upon and extrapolates from earlier knowledge, theories, thoughts, and concepts. In 1973, philosopher Hartry Field criticized Kuhn's thesis of incommensurability. His analysis emphasized the indeterminacy of reference within unique theories. As an example, Field took the term "mass" and questioned the exact meaning of "mass" in post-modern relativistic physics. Through his work, he found that "mass" had two definitions: (1) relativistic mass and (2) "real" mass. The former was defined by mass equaling the total energy of the particle divided by the speed of light squared, whereas the latter was defined by the mass of a particle equaling the non-kinetic energy of a particle divided by the speed of light squared. Field then projected his findings onto Newtonian dynamics, thus formulating two hypotheses: (1) mass denotes relativistic mass and (2) mass denotes "real" mass. Field concluded that it would be impossible to decide which of these two hypotheses is true since, prior to Einstein's theory of relativity, "mass" was referentially indeterminate; that is, mass was understood to be absolute. However, Field argues that "mass" in pre-Einsteinian physics meant something different than it means now. Therefore, a problem existed not within the meaning or interpretation of "mass," but within its reference.[6] Given Field's criticism, it can be seen that Kuhn's belief that paradigms are incommensurable is far too radical.

Finally, Kuhn's use of an evolutionary metaphor to explain science's pursuit of truth is troublesome. Kuhn's discussion of scientific progress and contention that science does not proceed to any predetermined truth is highly provocative. He maintains that science progresses as scientific theories become better articulated to accord with nature—that is, the solving of puzzles given by the working paradigm. Therefore, Kuhn's notion of progress seems indicative of the belief that scientists are able to revise their theories, thoughts, and concepts to generate more accurate representations of nature, thereby approaching some sort of truth. Nonetheless, Kuhn used an evolutionary metaphor to illustrate his argument. Applying Darwinian gradualism—a slow and gradual mode of evolution that occurs through natural selection (modification of existing species over a long period of time)—is seemingly antithetical to Kuhn's overarching argument about scientific progress. As Kuhn correctly notes, biological evolution is not Lamarckian in form—that is, biological evolution does not "progress" towards a directed goal. Kuhn likens the scientific process to that of Darwinian view of phyletic gradualism. However, he fails to take into consideration that unlike biological evolution, science *is* Lamarckian in form. Science *is* goal-oriented. Science *is* constantly improving.[7] Science is teleological but Darwinian evolution is not. Kuhn's incommensurability theory cannot adequately respond to the question posed at the beginning of this essay (i.e. "What is the nature of scientific advancement and progression?").

By exposing the apparent weaknesses within Kuhn's position of the nature of scientific development, it is evident that, due to the dynamic and cumulative nature of science, Kuhn's concept of paradigm is unrealistic, his belief that paradigms are incommensurable is too radical, and his use of an evolutionary metaphor to better illustrate his notion of scientific progress is troublesome.

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[6] Hartry Field, "Theory Change and the Indeterminacy of Reference," *The Journal of Philosophy,* 70 (14): 462-481.

[7] Lawrence Eng, "The Accidental Rebel: Thomas Kuhn and The Structure of Revolutions," (2001), <http://www.cjas.org/~leng/kuhn.pdf>.

IV. Science as a Greater Integer Function of Punctuated Equilibrium

Quickly reviewing what has been established thus far, (1) Kuhn claimed that science progressed towards no truth and only advanced through alternating practices of normal and revolutionary science—the adoption and abandonment of paradigms, (2) Kuhn's concept of paradigm is unrealistic, (3) Kuhn's belief that paradigms are incommensurable is too radical, and (4) Kuhn's use of an evolutionary metaphor to illustrate his notion of scientific progress is troublesome.

Everything that has been covered to date explains and critiques Kuhn's incommensurability theory on the nature of scientific progress as he inadequately attempts to respond to the questions posed at the beginning of this essay (i.e. What is the nature of science of scientific advancement and progression?). My answer to this question is to liken scientific progression to a greatest integer function of punctuated equilibrium. Thus, I attempt to reinstate the long-established notion that science is not a diminishing discipline but is, instead, a cumulative discipline.

Recall the greatest integer function (<u>Diagram 1.0</u>) introduced in the preface of this paper—a discontinuous piece-wise function that resembles a series of steps. The closed point indicates that the point is part of the graph, while the open point indicates that the point is not a part of the graph. The greatest integer function (step function) depicts a *cumulative* distribution of a random variable.

Recall, again, Eldredge and Gould's theory of punctuated equilibrium. This evolutionary theory depicts long periods of stasis interrupted by relatively brief periods of sudden change. The theory of punctuated equilibrium, as seen in <u>Diagram 1.1</u>, also resembles a series of steps that depicts a *cumulative* distribution of evolutionary change.

Now both <u>Diagram 1.0</u> and <u>Diagram 1.1</u> depict these individual steps on a greater, more general staircase, called *progress*. By specifying the staircase to a certain type of progress (i.e. scientific progress), the apparent cumulative development of science could be visually understood. However, as Kuhn argues, paradigms are incommensurable. The translation of concepts and meaning from paradigm to paradigm becomes distorted. The likening of science to a greater integer function/step function accommodates the supposed loss of meaning of concepts from older paradigms to newer paradigms, as the transition from each "step" involves the inclusion of all previous terms and concepts from the previous paradigm to the next paradigm (as represented by the closed point), leaving the reference point (open point) to be jettisoned. At the point of discontinuity, the only item of the previous paradigm left behind is the referential marker that indicated the point in time of the paradigmatic shift of theory.

The use of Darwinian gradualism as Kuhn's evolutionary metaphor for his interpretation of scientific progress was troublesome. However, perhaps Kuhn's desire to allude to an evolutionary metaphor to illustrate his notion of scientific progress was not so troublesome; perhaps Kuhn just used the wrong evolutionary theory.

Likening scientific progress to Eldredge and Gould's punctuated equilibrium theory is a better evolutionary metaphor for Kuhn's notion of scientific progress. As Eldredge and Gould contend, alternating long periods of environmental stasis and relatively brief stages of environmental change drive evolution. Kuhn maintains that scientific research in normal science adheres to a specific paradigm. The long periods of environmental stasis in Diagram 1.1 could theoretically represent Kuhn's periods of normal science working in adherence to a specific paradigm. The relatively brief stages of environmental change could represent Kuhn's revolutionary science working on transitioning from an old paradigm to a newly formulated one. Thus, the alternating periods of environmental stasis and brief stages of environmental change could represent Kuhn's idea of a mature science working in alternating periods of normal and revolutionary science.

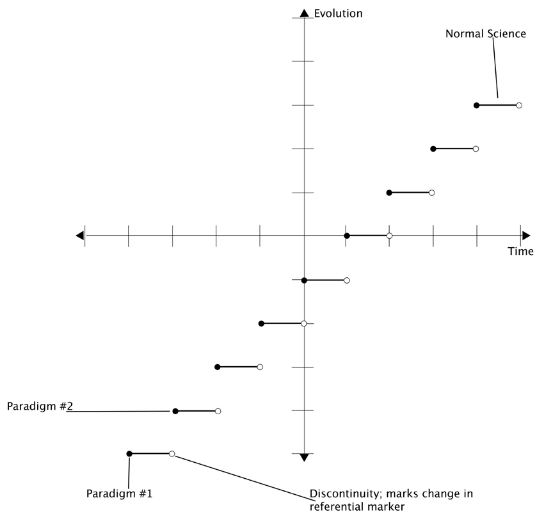
Putting Kuhn's work in context, the 1960's documented a wave of social revolutions. It is not surprising that a book as radical as Kuhn's gained much attention from academia. Radical movements of social change were occurring during the time Kuhn's *Structures* was published. Revolts against conservative norms and social conformity were occurring; the feminist movement was gaining momentum; the gay rights movement was taking flight; the Hispanic and Chicano Movement was taking place; and the African-American Civil Rights Movement was well underway. The modern West (particularly the U.S.) was in the midst of what Kuhn called a "crisis." It is not surprising that Kuhn's *Structures* was published during these happenings. As previously mentioned, prior to Kuhn's publication, science was always perceived to be a cumulative, objective, and rational discipline.

Scientists and philosophers of science prior to Kuhn practiced what Kuhn would describe as "normal science," functioning under a certain "paradigm." Likening the aforementioned to the theory of punctuated equilibrium, it could be argued that prior to Kuhn, science and philosophy of science were both stuck in a long period of "stasis." The publication and circulation of Kuhn's *Structures* marked the "revolution" or "abrupt change" during the "stasis" of the long adhered to "paradigm." The "crisis" preceding the "revolution" could have been the social revolution of the U.S., during *Structures* debut. By likening the stable environment pre-1960's (pre-publication of Kuhn's work) to the stable environment stasis in the punctuated equilibrium theory as depicted in Diagram 1.1, and by likening the abruptly changing environment of Kuhn's time to the abruptly changing environment that causes speciation in the punctuated equilibrium theory, Kuhn's work becomes contextualized and metaphorically understood through Eldredge and Gould's evolutionary theory.

Combining the greatest integer function with Eldredge and Gould's theory of punctuated equilibrium could allow for a reformulation of Kuhn's initial evolutionary metaphor that better explains his notion of scientific progress and simultaneously depicts science as a cumulative discipline.

Each closed circle on the graph represents a paradigm or scientific discovery, whereas each open circle on the graph (discontinuity) represents a referential marker indicating the point in time at which a new paradigm was deemed necessary due to the accumulation of scientific anomalies in the previous paradigm. Each line segment connecting the paradigm to the discontinuity represents a period in which normal science had been practiced. However, unlike the punctuated equilibrium theory where each horizontal line segment represented a period of environmental stasis, each line segment depicted in Diagram 4.0 represents a multitude of active particles that behave in a continuous, linear oscillation akin to the behavior of waves and particles as postulated by the wave-particle duality theory. These oscillating particles are representing the constant revisions—minute and enormous—made within the practice of science. Diagram 4.0 represents science as a cumulative process that pictorially documents the progress of science and visually depicts the transformation, revolution, and improvement of the discipline of science through the use of a greater integer function of punctuated equilibriums.

Diagram 4.0 The Progress Staircase of Science as a Greater Integer Function of Punctuated Equilibrium



V. Conclusion

As Paul Teller states, "We start with inexact, prescientific, representational tools. Using these we solve certain problems by correcting, extending, and refining our means of representation, which are then absorbed back into the overall conceptual toolkit."[8] Representations in science are cumulative. The nature of science is dynamic. Science is endemic to our society because it *is* Lamarckian in form and it *is* in the constant pursuit of truth. The process of improvement in accuracy and precision is continuous.[9] The provisional nature of science is what grants it the ability to continually improve and progress. The more it revises, changes, and improves, the more accurate and precise the discipline will become. Controversy and discussion of competing theories and facts is a sign that good scientific advancements are in development. Having utilized a mathematical and evolutionary

[8] Paul Teller, "Representations in Science," *The Routledge Companion to Philosophy of Science*, eds. Stathis Psillos and Martin Curd (New York: Routledge, 2008): 440.
[9] Ibid., 440.

metaphor to illustrate and reformulate Kuhn's notion of scientific progress, I have attempted to respond to the question posed at the beginning of this essay by reinstating the long-established notion that science is *not* a diminishing discipline but is, instead, a *cumulative* discipline. ❖